RAC 1452-7
15 May 1965

User's Manual
for
PHASE V: STRESS ANALYSIS OF A DOUBLY-CURVED SKIN WITH A FLARED NOZZLE PORT

Contract NAS 8-2698
(RAC 1452-7)

Submitted to
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
George C. Marshall Space Flight Center
Huntsville, Alabama

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FOREWORD

This report was prepared by Dr. I. U. Ojalvo of Republic Aviation Corporation, Farmingdale, New York, under Contract No. NAS 8-2698, "Stress Analysis of a Doubly-Curved Skin with a Flared Nozzle Port."

The work was administered under the direction of Mr. David Hoppers of the Manufacturing Engineering Laboratory through Mr. Norman Schlemmer of the Propulsion and Vehicle Engineering Laboratory of the George C. Marshall Space Flight Center.

The Republic Program Manager is Dr. R. S. Levy.

ACKNOWLEDGEMENTS

The writer gratefully acknowledges the contribution of Mr. N. Levine for supervising the entire digital programming effort.

TABLE OF CONTENTS

SECTION I

INTRODUCTION

26048 ABSTRACT

The operation of a digital program to determine the stresses and deflections of flared nozzles in doubly-curved thin, shallow, domes is described. The analysis and computer flow charts upon which the machine program is based are presented in Reference 1.*

The structural problem is idealized as a flared shell of revolution with axis normal to a thin, shallow, parent shell of double curvature (see Figure 1). The mid-surfaces of the two shells are assumed to mate at a common intersecting circle and the entire configuration is subjected to internal pressurization and membrane edge forces.

The sign convention used for deflections, rotations, forces, and moments is presented in Figure 2.

The computer program consists of six separate parts which may be run in one machine pass but must be placed in consecutive numerical order, i.e. Parts 1, 2, ..., 6. Input for Part 1 follows the program deck for Part 1. Input for Part 2 follows the program deck for Part 2, etc., to Part 6. The input for Parts 2, 3, and 5 are identical. Part 6 differs in only one input card, as described in Section V. Therefore, input instructions for Parts 1, 2, and 3 only, are given in detail.

Author

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*1. "Annual Summary Report for Phase V: Stress Analysis of a Doubly-Curved Skin with a Flared Nozzle Port," Republic Aviation Corporation Report No. RAC 1452-6, 15 May 1965.

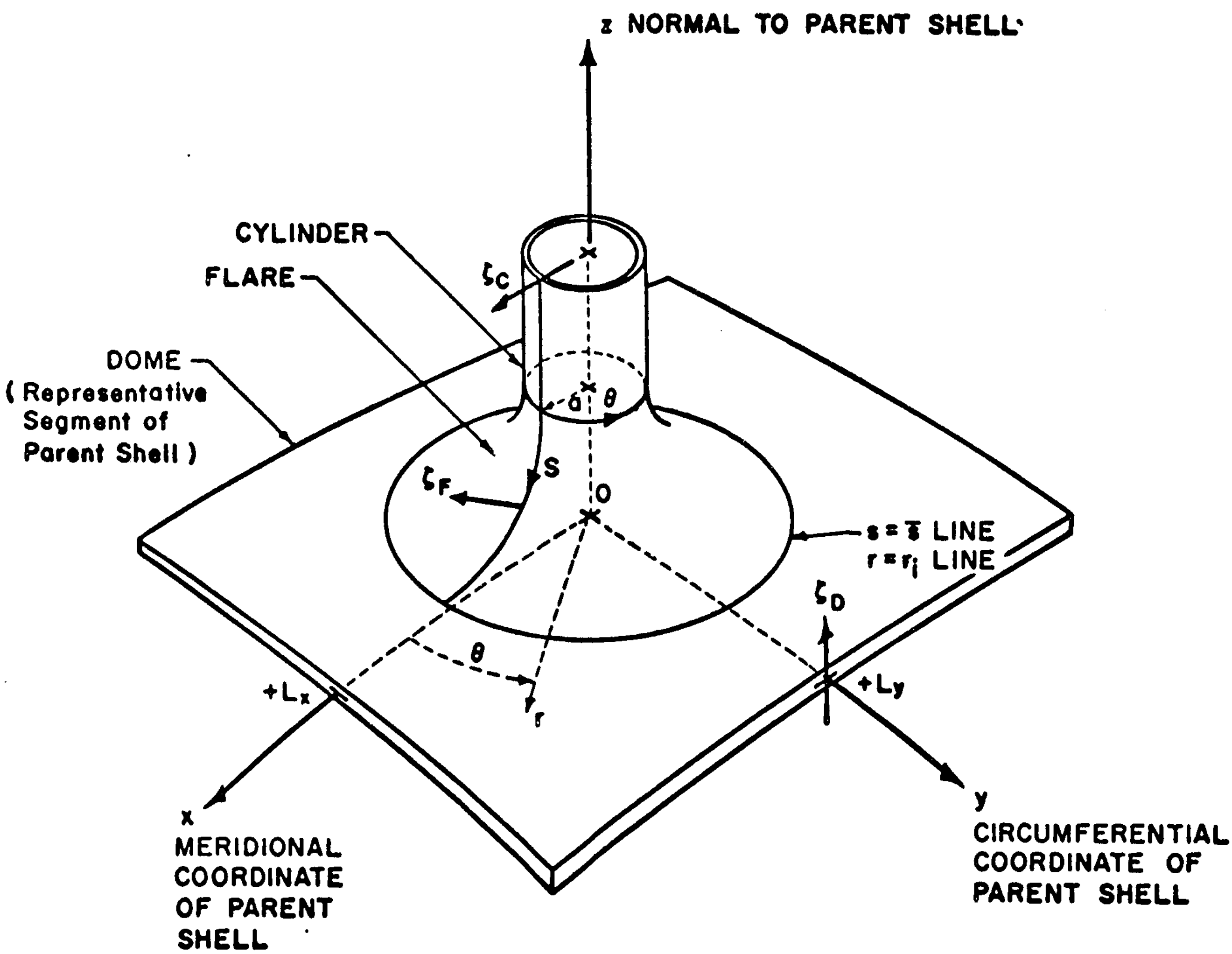


Figure 1. Coordinate Systems

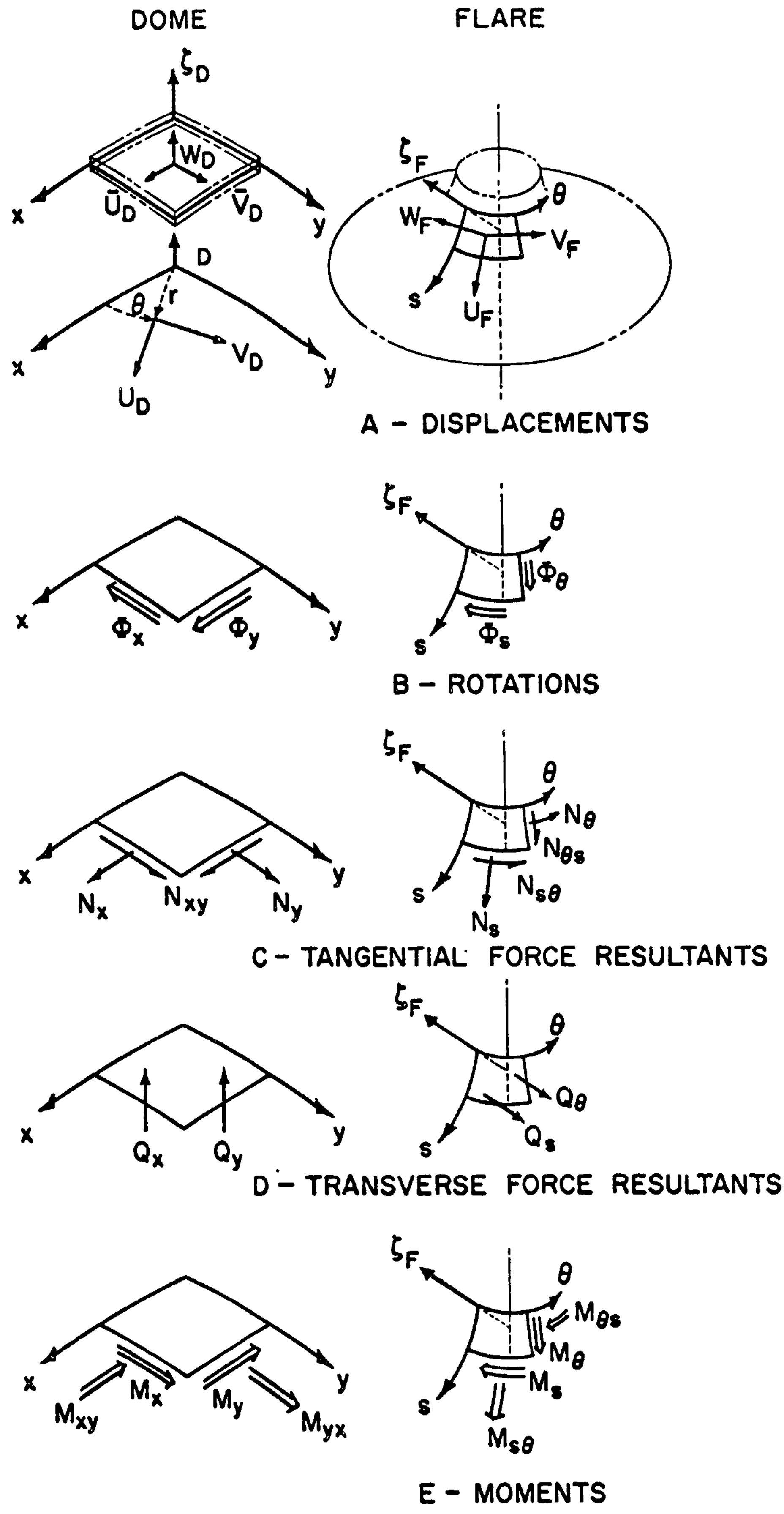


Figure 2. Sign Conventions

SECTION II

INPUT QUANTITIES

A. CYLINDER-FLARE GEOMETRY

The geometric data describing the cylinder-flare configuration may be defined by a series of input coordinates ($r_k{}'$, $z_k{}'$), thicknesses ($t_k{}'$), and meridional curvatures ($\omega_k{}'$), (see Figure 3a), or as a quarter of an ellipse which is tangent to the cylinder and dome (see Figure 3b).

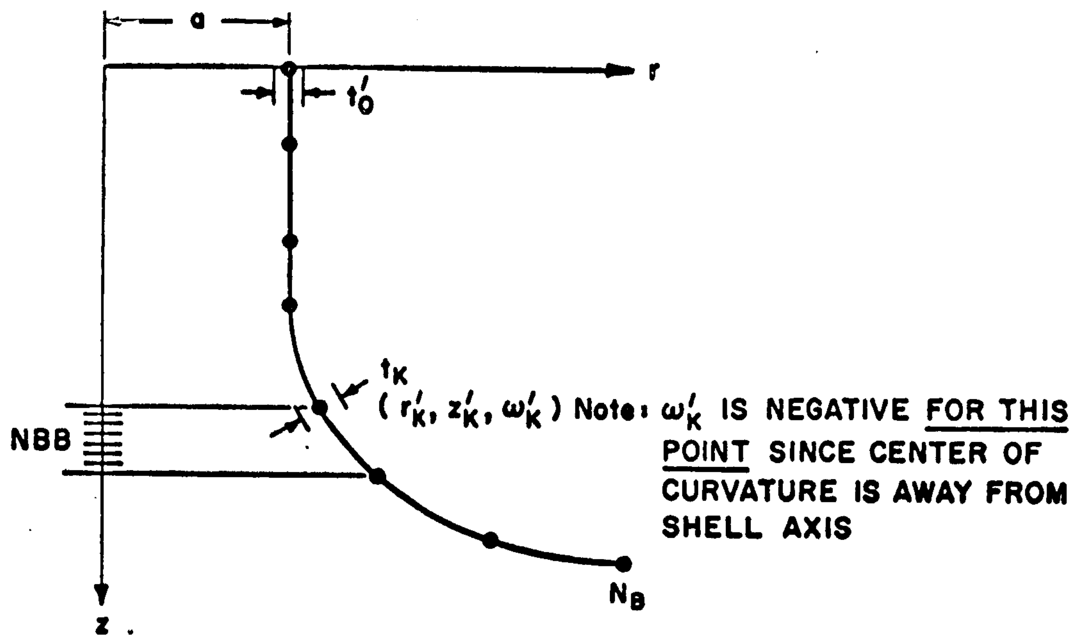
B. DOME GEOMETRY AND POINTS MATCHED

Specification of the dome geometry requires the principal dome curvatures $\frac{1}{R_x}$, $\frac{1}{R_y}$, the dome boundaries (L_x, L_y, r_i), and the thickness (t_D), of the dome. R_x is the radius of curvature of the dome along the x axis of Figure 4, and R_y is the radius of curvature of the dome along the y axis.

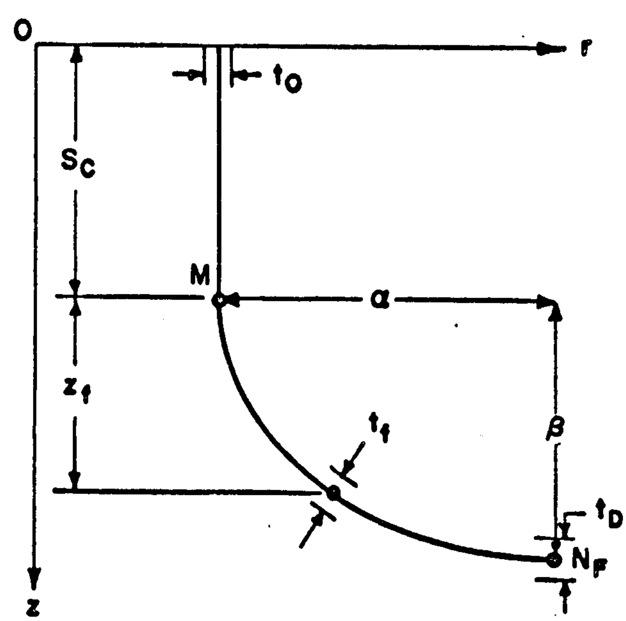
The points at which boundary conditions are matched are indicated by dots in Figure 4.

NDY is the number of points along the y axis at which $\bar{Q}_x$ is set equal to zero. The YD are the ordinates of these points. In a similar manner, NDX and NDXP establish the points at which $\bar{Q}_y$ is set equal to zero. These are necessary symmetry conditions which are not automatically imposed by the dome trial functions, and one algebraic equation is developed for each point selected.

NDXP is the number of points along $y = L_y$ at which membrane boundary conditions are satisfied and XPD are the specific points. Similarly, NDYP and YPD relate to membrane conditions along $x = L_x$. Since there are four conditions to be satisfied, 4(NDXP + NDYP) is the number of algebraic equations specified by these points.



a. Input Option 1



b. Input Option 2

Figure 3. Cylinder-Flare Input Geometry

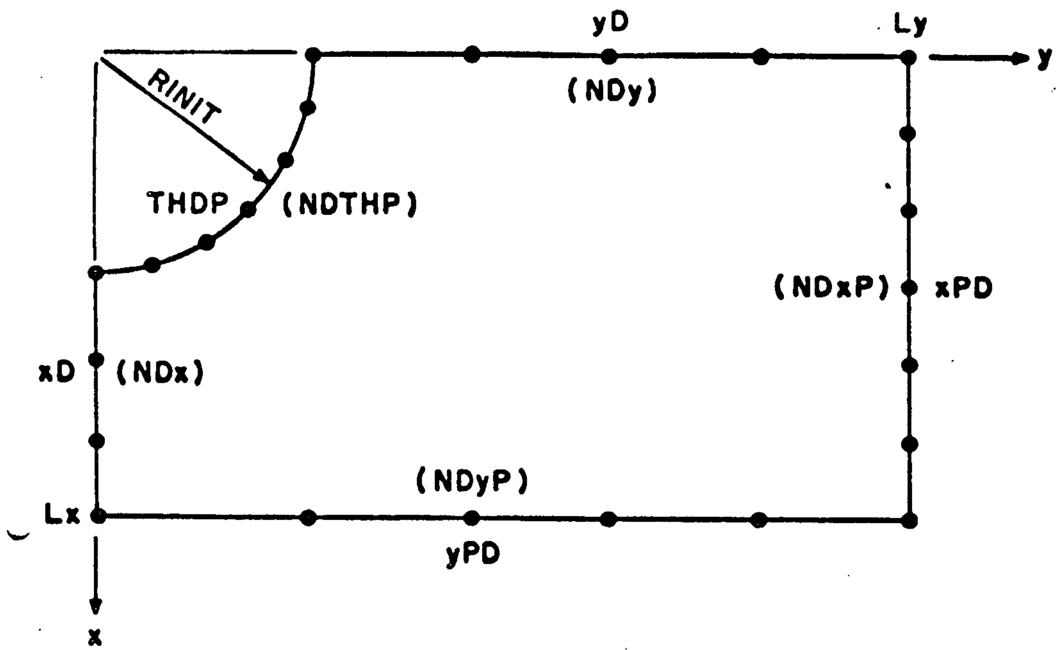


Figure 4. Points at Which Boundary Conditions are Matched

NDTHP is the number of points along the flare-dome intersection at which compatibility and equilibrium are satisfied. The specific points are defined by the THDP. There are eight algebraic equations for each THDP. Thus, 8(NDTHP) equations result from these points.

There are

(NDX + NDY) + 4(NDXP + NDYP) + 8(NDTHP)

equations in all.

I + 1 and J + 1 are the number of constants introduced by the dome solution and $2\bar{n} + 3$ are the number of flare constants, where I, J, and $\bar{n}$ are input integers ($\bar{n}$ must be even)

$$(I + 1) + (J + 1) + (2\bar{n} + 3)$$

constants in all.

To obtain a unique system of point-matched algebraic constants, it is necessary that (NDX + NDY) + 4(NDXP + NDYP) + 8(NDTHP) = (I + 1) + (J + 1) + ($2\bar{n}$ + 3). However, a least squares solution is obtained if the number of equations is greater than the number of unknowns.

C. OUTPUT STATIONS

Flare stresses and deflections are computed for all the meridional finite difference stations in the flare and at specified angles θ. The θ angles are determined by θ_0, $\Delta\theta$, and θ_ℓ. Thus the angles selected become

$$\theta_0,\ \theta_0 + \Delta\theta,\ \theta_0 + 2\Delta\theta,\ \ldots.,\ \theta_\ell$$

Dome stresses and deflections are computed for the same angles as given for the flare and at the radial coordinates r as determined by r_i, Δr, L_x, and L_y (see Figure 5).

In addition, it is possible to obtain cartesian coordinate results for the dome by specifying the appropriate option. The input quantities which select these stations are x_0, Δx, and Δy as shown in Figure 6.

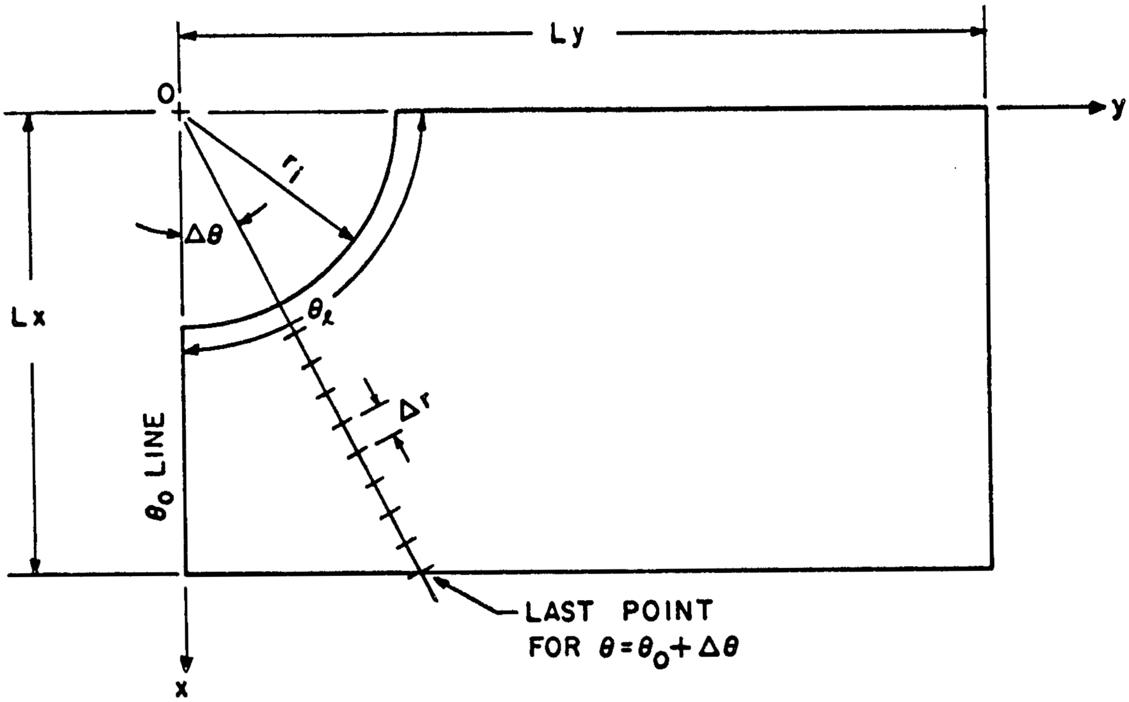


Figure 5. Polar Output Stations for Dome

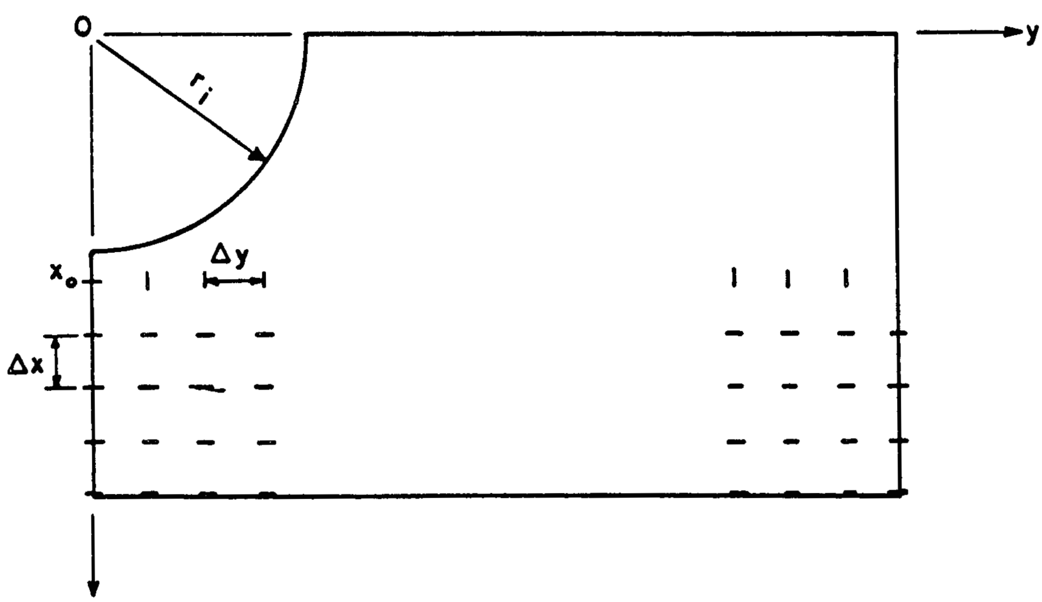


Figure 6. Cartesian Dome Output Stations

SECTION III

INPUT NOMENCLATURE

Physical Symbols	Dimensional Units	Program Symbols	Physical Description
E	psi	EF, ED, ELAS	Modulus of elasticity; assumed the same for dome-flare-cylinder configuration
γ		NU, NUD	Poisson's ratio
p	psi	PSI	Internal pressure
r_c	in.	RC	Characteristic radius
t_c	in.	TC	Characteristic radius
σ_c	psi	SIGC	Characteristic stress
$\bar{n}$		NBAR	Maximum Fourier index for flare

If NC option 2 = 1, use the following flare input geometry. (Reference Figure 9a)

Physical Symbols	Dimensional Units	Program Symbols	Physical Description
$\bar{N}$		NB	Number of flare input coordinate points
$\bar{\bar{N}}$		NBB	Number of finite-difference intervals into which segment between successive input points is subdivided
N_F		NF	Number of flare finite difference intervals = $\bar{N} \cdot \bar{\bar{N}}$
a	in.	A, ACR	Radius of cylinder
r'_k	in.	RPK	Radii of cylinder-flare input points
z'_k	in.	ZPK	Vertical coordinate of cylinder-flare input points
t'_o	in.	TZP	Thickness at top of cylinder
ω'_o	in.$^{-1}$	OMZP	Curvature at top of cylinder
ω'_k	in.$^{-1}$	OMPK	Curvature at cylinder-flare input points

If NC Option 2 = 0, use the following flare input geometry. (Reference Figure 9b)

Physical Symbols	Dimensional Units	Program Symbols	Physical Description
α	in.	ALPHA	Horizontal semi-axis (ellipse) for flare geometry
β	in.	BETA	Vertical semi-axis (ellipse) for flare geometry
z_F	in.	ZF	Flare thickness parameter; adjusted so quadratic flare thickness approximates desired thickness variation
S_c	in.	SC	Cylinder length
$\bar{t}_D$	in.	TTDD	Flare thickness parameter
L_x	in.	LX	Dome half-span in x direction
L_y	in.	LY	Dome half-span in y direction
t_D	in.	TD	Dome thickness
$1/R_x$	$in.^{-1}$	RATIO X	Dome curvature in x direction
$1/R_y$	$in.^{-1}$	RATIO Y	Dome curvature in y direction
I		ID	Maximum index on series which decays exponentially in y direction
J		JD	Maximum index on series which decays exponentially in x direction
NDY		NDY	Number of points matched along y axis
NDX		NDX	Number of points matched along x axis
NDYP		NDYP	Number of points matched along $y = L_y$
NDXP		NDXP	Number of points matched along $x = L_x$
NDTHP		NDTHP	Number of points matched along $r = r_i$
r_i	in.	RINIT	Radius of flare-dome intersection
Δr (or ΔR)	in.	DELTAR	Radial increment for output stations for stress and deflection computation
x_o	in.	XO	Cartesian output stations
Δx	in.	DELTAX	Increment for cartesian output stations

Physical Symbols	Dimensional Units	Program Symbols	Physical Description
Δy	in.	DELTAY	Increment for cartesian output stations
θ_o	degrees	THETAO	Polar output stations
$\Delta\theta$	degrees	DTHETA	Polar increment for output stations
θ_ℓ	degrees	THLAST	Final angle for polar output stations

SECTION IV
INPUT PROCEDURE

The order for inputting the required information on standard IBM cards is as shown in the tables on the following pages.

Key to abbreviations:

II	-	Input Indicator
IC	-	Integer Constant
SN	-	See Nomenclature
FP	-	Floating Point Format
FXP	-	Fixed Point Format

FLARE INPUT - PART 1

Card Type	Columns	Quantity	Format	Description
1	1 thru 72	Title	A - conversion	Program title and date
2	1 thru 72	NC	IC	Intermediate printout flags are located in columns 1 thru 72. If no intermediate printout is desired, leave columns 1 thru 72 blank, i. e., $NC(i) \begin{cases} = 0 & \text{no print out} \\ = 1 & \text{printout} \end{cases}$
3	1 thru 72	NC	IC	Same as Card 2 except that it governs printout for flags 73 thru 144.
4	1 thru 69	NC	IC	Path decision flags (options are located in columns 2 and 69): $NC(2) \begin{cases} = 0 & \text{ellipse curve fit for flare} \\ = 1 & \text{cubic curve fit for flare} \end{cases}$ $NC(69) \begin{cases} = 0 & \text{does not compute additional dome output in cartesian coordinates} \\ = 1 & \text{does compute additional dome output in cartesian coordinates} \end{cases}$
5	5	1	IC	II
6	1 thru 10	E	FP	SN
	11 thru 20	γ		
7	5	2	IC	II
8	1 thru 10	p	FP	SN
9	5	3	IC	II
10	1 thru 10	R_c	FP	SN
	11 thru 20	t_c	FP	SN
	21 thru 30	σ_c	FP	SN
11	5	4	IC	II

Card Type	Columns	Quantity	Format	Description
12	6 thru 10	NBAR	FXP - right adjusted	SN
Use card types 13 thru 22 only if card type 4 has a 1 (i.e., NC(2) = 1), in column 2. If NC(2) = 0, use card types 23 thru 29.				
13	5	5	IC	II
14	1 thru 5	$\bar{N}$	FXP	SN
	6 thru 10	$\bar{\bar{N}}$	FXP	SN
15	5	6	IC	II
16	1 thru 10	a	FP	SN
	11 thru 20	r_1'		Up to 7 entries per card
	.			
	.			
	.			
	.			
	61 thru 70	r_6'		
	1 thru 10	r_7'		
	.			
	.			
	.			
	.	$r_{\bar{N}}'$		
17	5	7	IC	II
18	1 thru 10	z_1'	FP	SN
	.			Up to 7 entries per card
	.			
	.			
	.			
	61 thru 70	z_7'		

Card Type	Columns	Quantity	Format	Description
	1 thru 10	z'_8	FP	SN
	.			Up to 7 entries per card
	.			
	.			
	.			
	.	$z'_{\bar{N}}$		
19	5	8	IC	II
20	1 thru 10	t'_o	FP	SN
	11 thru 20	t'_1		Up to 7 entries per card
	.			
	.	$t'_{\bar{N}}$		
21	5	9	IC	II
22				Blank Card

Use card types 23 thru 29 only if card type 4 has a zero (i. e., NC(2) = 0) in column 2.

23	4 and 5	10	IC	II
24	1 thru 10	a	FP	SN
	11 thru 20	p	FP	SN
	21 thru 30	α	FP	SN
	31 thru 40	β	FP	SN
	41 thru 50	t_f	FP	SN
	51 thru 60	S_c	FP	SN
	61 thru 70	t_o	FP	SN

Card Type	Columns	Quantity	Format	Description
25	1 thru 10	$\bar{t}_D$	FP	SN
26	4 and 5	11	IC	II
27				Blank Card
28	4 and 5	12	IC	II
29	1 thru 5	M	FXP	SN
	6 thru 10	NF	FXP	SN
30	4 and 5	13	IC	II
31	1 thru 10	10^{-8}	FP	Control for matrices inverted in flare solution.

Use card types 32 and 33 only if NC(2) = 1 in column 2 of card type 4.

Card Type	Columns	Quantity	Format	Description
32	4 and 5	14	IC	II
33	1 thru 10	ω_o'	FP	SN
	11 thru 20	ω_1'	FP	SN
		.		
		.		
		.		
		$\omega_{\bar{N}}'$	FP	SN
34	4 and 5	15	IC	II

DOME INPUT - PARTS 2, 4, 5 and 6

Card Type	Columns	Quantity	Format	Description
1	1 thru 72	Title	A - conversion	Program title and date
2	1 thru 72	NC	IC	Intermediate printout flags are located in columns 1 thru 72. Place a 1 in column 1 for part 6 only.
3	1 thru 72	NC	IC	Intermediate printout flags in colums 1 thru 72.
4	1 thru 72	NC	IC	Place a 2 in column 70 for part 2 input only.
5	1 thru 10	L_x	FP	SN
	11 thru 20	L_y		
	21 thru 30	t_D		
	31 thru 40	R_x^{-1}		
	41 thru 50	R_y^{-1}		
	51 thru 60	I		
	61 thru 70	J		
6	1 thru 10	E		
	31 thru 40	γ		
7	1 thru 10	NDY		Number of $\bar{Q}_x = 0$ points to be matched along y axis.
	11 thru 20	NDX		Number of $\bar{Q}_y = 0$ points to be matched along x axis.
	21 thru 30	NDYP		$x = L_x$ number of membrane boundary condition points.
	31 thru 40	NDXP		$y = L_y$ number of membrane boundary condition points
	51 thru 60	NDTHP		$r = r_i$ number of compatability points matched.
	61 thru 70	$\bar{n}$		Maximum Fourier index on flare solutions
8	1 thru 10	N_F	FP	Number of finite difference intervals. Must = $\bar{N} \cdot \bar{\bar{N}}$ if NOPT(2) = 1 in card type 4 of part 1 input.

Card Type	Columns	Quantity	Format	Description
9	1 thru 10	p	FP	SN
	11 thru 20	R_c		
	21 thru 30	t_c		
	31 thru 40	σ_c		
	51 thru 60	a		
10	1 thru 10	r_i		
	11 thru 20	Δ_r		
11	1 thru 10	x_o		
	11 thru 20	Δx		Only if NOPT 69 $\neq$ 0
	21 thru 30	Δy		Otherwise Blank Card
12	1 thru 10	θ_o		SN
	11 thru 20	Δ_θ		
	21 thru 30	θ_ℓ		
13	1 thru 10	$x D_1$	FP	Stations corresponding to NDX
	11 thru 21	xD_2		
		.		
		.		
		.		
		$xD_{(NDX)}$		
14		yD_1	FP	Stations corresponding to NDY

Card Type	Columns	Quantity	Format	Description
15		XPD		Stations corresponding to NDXP
16		YPD		Stations corresponding to NDYP
17				Blank Card
18		THPD		Stations corresponding to NDTHP

INPUT - PART 3

1	1	2	IC	II
2	29 and 30	10	FXP	Number of iterations of point matching system.

SECTION V

SAMPLE INPUT

For purposes of illustrating the method of inputting data to the program, consider the photoelastic test model N-4A of Reference 2. The nozzle-sphere geometry, shown and dimensioned in Figure 7a, was subjected to internal pressure which yielded photoelastic patterns that were subsequently measured. The experimental stress results are reproduced in Figure 7b. Poisson's ratio for the epoxy resin test specimen was approximately 0.45.

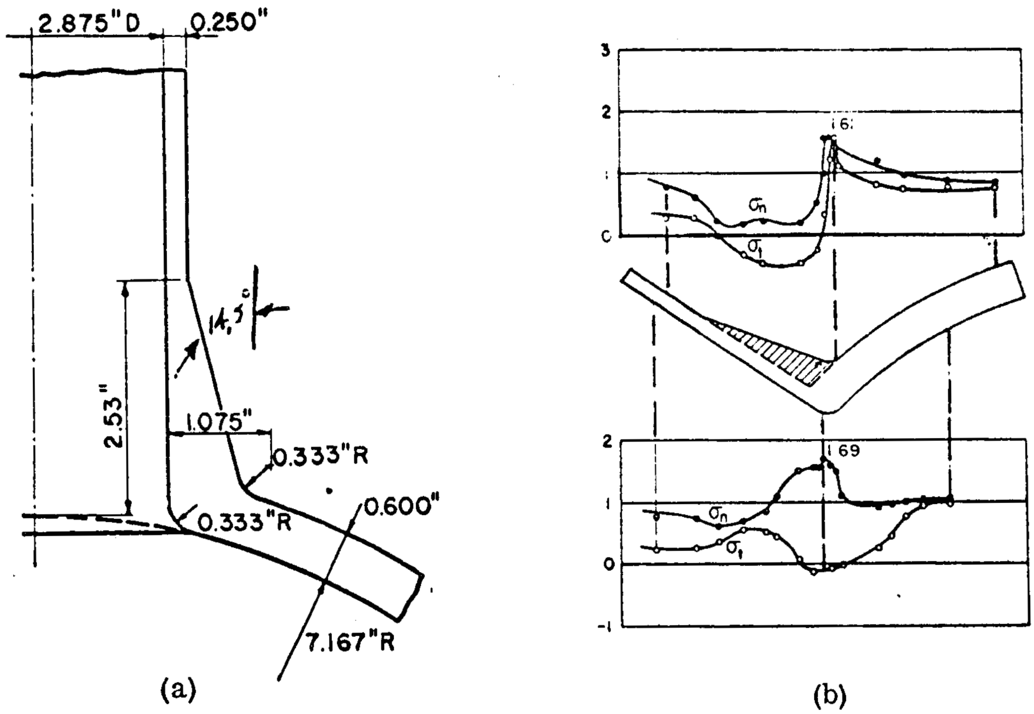


Figure 7. Photoelastic Test Model N-4A of Reference 2

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2. Taylor, L. E., Lind, N. C., and Schweiker, J. W., "A Three-Dimensional Photoelastic Study of Stresses Around Reinforced Outlets in Pressure Vessels", Welding Research Council Bulletin No. 51, June 1959.

It is now necessary to idealize the nozzle as a shell and thus determine the middle surface and thickness properties z_i', r_k', t_k', ω_k'. A tabulation of these is given below:

Point on Figure 8	z_k'	r_k'	t_k'	ω_k'
0	0	a = 1.5625	.250	0
1	2.50	a = 1.5625	.250	0
2	4.41	a = 1.5625	.250	0
3	4.96	a = 1.5625	.250	0
4	5.43	a = 1.6075	.375	0
5	5.84	a = 1.6600	.450	0
6	6.87	a = 1.7600	.650	0
7	7.25	a = 1.8200	.830	-2.2222
8	7.34	a = 1.8700	.850	-4.35
9	7.53	a = 2.0700	.680	.174
10	7.61	a = 2.3200	.600	.174

To determine the stresses and deflections analytically, input the following cards.

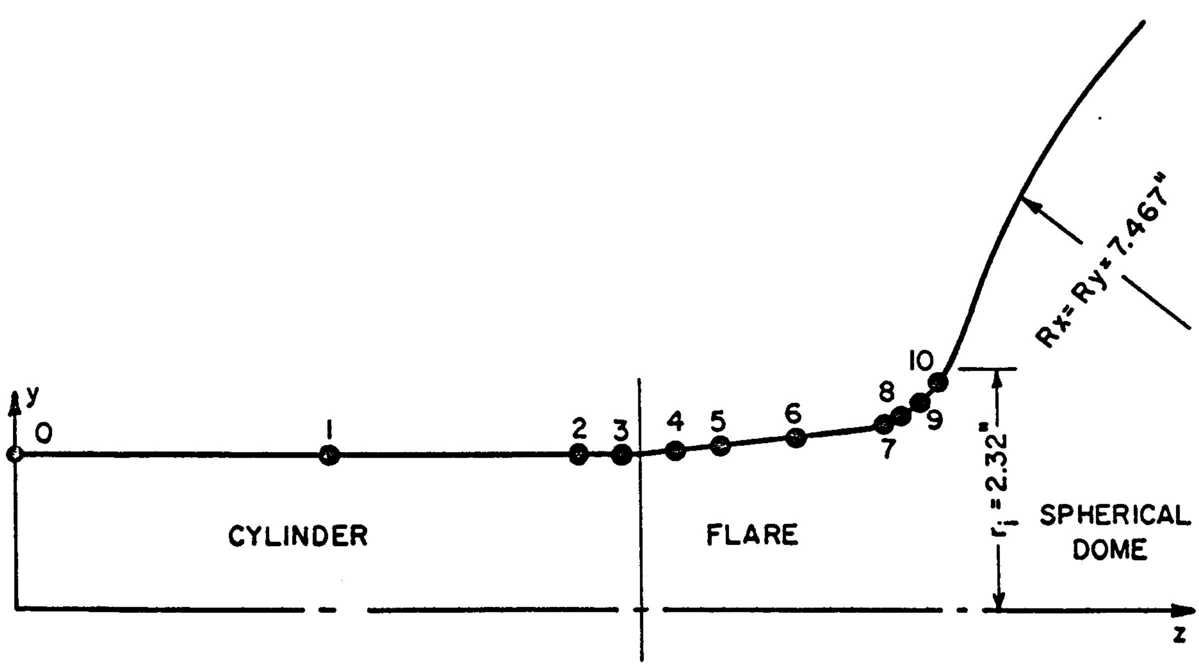

Figure 8. Flare Input Geometry

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	OPERATION NOTES	WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
1	Title, Part 1	FLARED NO	ZZLE N-4A	TEST CASE	CAPT I FL	ARE			
2	BLANK CARD								
3	BLANK CARD								
4	CUBIC CURVE FIT OPTION								
5	Input Indicator								
6	E, ν		.45						
7	Input Indicator	2							
8	F	.161							
9	input Indicator	3							
10	R_c, t_c, σ_c	1.	1.	1.					
11	input indicator	4							
12	$\bar{n}$	6							
13	Input Indicator	5							
14	$\bar{N}$, $\bar{\bar{N}}$ FIXED POINT	10 0							
15	input Indicator	6							
16	u, r′(1), … r′(6)	1.5625	1.5625	1.5625	1.5625	1.6075	1.66	1.76	
17	r′(7), … r′(10)	1.82	1.87	2.07	2.32				
18	Input Indicator	7							
19	z′(1), … z′(7)	2.5	4.41	4.96	5.43	5.84	6.67	7.25	
20	z′(8), z′(9), z′(10)	7.39	7.53	7.61					
21	input Indicator	8							
22	t_0, t_i, … t′(6)	.25	.25	.25	.25	.375	.45	.65	
23	t′(7), … t′(10)	.85	.85	.68	.60				
24	Input Indicator	4							
25	BLANK CARD								
26	Input Indicator	19							
27	Control on Flare Matrices Inverted	1. -8							
28	Input Indicator	14							
29	ω_0, ω′(1), … ω′(6)	0.	0.	0.	0.	0.	0.	0.	
30	ω(7), … ω(10)	-2.2222	-4.35	.174	.174				
E-1-206 REV. 3/56	Indicator Card	15							

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	OPERATION NOTES	WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
1	Title, PART 2	[illegible]	[illegible]	[illegible]	PART 2				
2	BLANK CARD, Place a 1 in Col 1 for Part 6 only								
3	BLANK CARD								
4	BLANK CARD								
5	[illegible]	5	5.	6	.134	.134	[illegible]	.24	
6	[illegible]	1	.45						
7	NDY, NDX, NDYP, NDXP, BLANK, [illegible]	4.	4.	3.	3.				
8	[illegible]	100.							
9	[illegible], BLANK, [illegible]	.161	1.	1.	1.		[illegible] 2.5		
10	[illegible]	2.32	1.						
11	BLANK [illegible]								
12	[illegible]	[illegible]	3.	50.					
13	XD(1), , XD(4)	2.32	3.32	4.32	5.				
14	YD(1), , YD(4)	2.32	3.32	4.32	5.				
15	XPD(1), , XPD(3)	0.	21.5	5.					
16	YPD(1), , YPD(3)	0.	21.5	5.					
17	BLANK CARD								
18	THPD(1), , THPD(4)	0.	30.	60.	90.				
19									
20									
21									
22									
23									
24									
25									
26									
27									
28									
29									
30									

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	OPERATION NOTES	WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
1	PART 3 - BLANK CARD								
2	Number of Solution Iterations			10					
3									
4									
5									
6	- PARTS 4 and 5 identical to								
7	Input of Part 2								
8	- PART 6 Identical to Input								
9	of Part 2 EXCEPT								
10	for a 1 in Column 1								
11	of CARD 2								
12									
13									
14									
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